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A (p, q) deformation of the universal enveloping superalgebra $U(osp(2/2))$

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Abstract. We investigate a two-parameter quantum deformation of the universal enveloping orthosymplectic superalgebra $U(osp(2/2))$ by extending the Faddeev–Reshetikhin–Takhtajan formalism to the supersymmetric case. It is shown that $U_{p,q}(osp(2/2))$ possesses a non-commutative, non-co-commutative Hopf-algebra structure. All the results are expressed in the standard form using quantum Chevalley basis.

There has been an enormous interest in quantum deformations of Lie groups and Lie algebras during the last couple of years. The next step in this direction has been to extend these ideas to supergroups and superalgebras [1–6]. The aim of the present investigation is to obtain a two-parameter deformation of the universal enveloping algebra of the orthosymplectic Lie superalgebra $osp(2/2)$. This is achieved by extending the basic considerations of the R-matrix approach proposed by the Leningrad school [7], to the supersymmetric case. We also find out the action of various maps on the generators of $U_{p,q}(osp(2/2))$ to show that it is indeed equipped with a Hopf algebra [8] structure which is non-commutative as well as non-co-commutative. Finally, all the super-commutation relations obtained in the R-matrix framework are transformed into the standard form with the help of the quantum analogues of the Chevalley generators.

The algebra of functions on a quantum supergroup is defined by the relations

$$\hat{R}\; T_1 T_2' = T_1 T_2' \,\hat{R} \tag{1}$$

where the matrix $\hat{R}$ is a solution of braid QYBE and corresponds to that particular quantum supergroup. T is the transformation matrix and

$$\begin{aligned} (T_1)^{ab}_{cd} &= (T\otimes I)^{ab}_{cd} = (-1)^{c(b+d)}\; T^a_c\; \delta^b_d \\ (T_2)^{ab}_{cd} &= (I\otimes T)^{ab}_{cd} = (-1)^{a(b+d)}\; T^b_d\; \delta^a_c \\ T_2' &= \mathcal{P}\; T_1\; \mathcal{P} \end{aligned} \tag{2}$$

where $\mathcal{P}$ is the super-permutation matrix

$$(\mathcal{P})^{ab}_{cd} = (-1)^{ab}\; \delta^a_d \delta^b_c\,. \tag{3}$$

However, for the case of orthosymplectic supergroups we need to impose additional conditions:

$$CT^{\rm st}\, C^{-1} T = TCT^{\rm st}\, C^{-1} = I\,. \tag{4}$$

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Here C is an antidiagonal metric and $T^{\rm st}$ is the supertranspose of T defined as

$$(T^i_j)^{\rm st} = (-1)^{j(i+j)}\, T^j_i\,. \tag{5}$$

Relation (4) is the modified form of the conditions given by FRT [7] for orthogonal and symplectic groups separately. So the algebra of functions on the orthosymplectic group is defined by relations (1) and (4) together.

Let us now consider a specific example of the quantum supergroup $OSp(2/2)$. The non-vanishing elements of the (16×16) $\hat{R}$ matrix are [9]

$$\begin{aligned}
&\hat{R}^{11}_{11} = \hat{R}^{44}_{44} = -(pq)^{-1/2} \qquad \hat{R}^{22}_{22} = \hat{R}^{33}_{33} = (pq)^{1/2}\\
&\hat{R}^{12}_{21} = \hat{R}^{31}_{13} = \hat{R}^{24}_{42} = \hat{R}^{43}_{34} = (p/q)^{1/2}\\
&\hat{R}^{21}_{12} = \hat{R}^{13}_{31} = \hat{R}^{42}_{24} = \hat{R}^{34}_{43} = (p/q)^{-1/2} \qquad \hat{R}^{12}_{12} = \hat{R}^{13}_{13} = \hat{R}^{24}_{24} = \hat{R}^{34}_{34} = (pq)^{1/2} - (pq)^{-1/2}\\
&\hat{R}^{32}_{23} = \hat{R}^{23}_{32} = (pq)^{-1/2} \qquad \hat{R}^{41}_{14} = \hat{R}^{14}_{41} = -(pq)^{1/2}\\
&\hat{R}^{14}_{14} = \left((pq)^{1/2} - (pq)^{-1/2}\right)(1-(pq)^{-1}) \qquad \hat{R}^{23}_{23} = \left((pq)^{1/2} - (pq)^{-1/2}\right)(1+(pq)^{-1})\\
&\hat{R}^{23}_{14} = \hat{R}^{14}_{23} = i(pq)^{-1}\left((pq)^{1/2} - (pq)^{-1/2}\right) \qquad \hat{R}^{14}_{32} = \hat{R}^{32}_{14} = -\mathrm{i}\left((pq)^{1/2} - (pq)^{-1/2}\right).
\end{aligned} \tag{6}$$

where p and q are two (complex) deformation parameters. $T \equiv (T^i_j)_{i,j=1,\ldots 4}$ is a 4×4 super-matrix acting on a quantum vector space generated by the elements x^1, x^2, x^3, x^4, where the indices $(1, 4)$ are fermionic and $(2, 3)$ are bosonic. The metric C in this case is 4×4 and given by [9]

$$C = \begin{pmatrix} 0 & 0 & 0 & \mathrm{i}(pq)^{-1/2} \\ 0 & 0 & (pq)^{-1/2} & 0 \\ 0 & -(pq)^{1/2} & 0 & 0 \\ \mathrm{i}(pq)^{1/2} & 0 & 0 & 0 \end{pmatrix}. \tag{7}$$

A crucial feature of quantum $OSp(2/2)$ is that it leaves the following constraint invariant:

$$x^t C x = \mathrm{i}(pq)^{1/2} x^4 x^1 - (pq)^{1/2} x^3 x^2 + (pq)^{-1/2} x^2 x^3 + \mathrm{i}(pq)^{-1/2} x^1 x^4 = 0\,. \tag{8}$$

The homogeneous quadratic part of the algebra is preserved in the usual way by virtue of the (super-) RTT relations (1) whereas the inhomogeneous part is left covariant by virtue of (4).

Thus we obtain the complete set of deformed commutation relations among the generators T^i_j of the functions on the quantum supergroup $OSp_{p,q}(2/2)$, by substituting the matrices $\hat{R}, T$ and C in (1) and (4). It turns out that the elements $(T^i_j)_{ij=1,\ldots 4},\ i \neq j'$ (where $1' = 4, 2' = 3$) behave as odd generators (since their squares vanish) and the rest as even.

Our main purpose in this letter is to obtain a deformation of the dual object i.e. the universal enveloping superalgebra $U\ osp(2/2)$. The generators are arranged in 4×4 upper and lower triangular matrices $L^{(\pm)} \equiv (L^{(\pm)}{}^i_j)_{i,j=1,\ldots 4}$ of regular functionals and obey the (super-) commutation relations

$$R_{21}\ L_1^{(\varepsilon_1)}\ L_2^{(\varepsilon_2)} = L_2^{(\varepsilon_2)}\ L_1^{(\varepsilon_1)}\ R_{21} \tag{9}$$

where $(\varepsilon_1, \varepsilon_2) = (+,+), (+,-), (-,-)$, $R_{21} = \mathcal{P} R_{12} \mathcal{P}$. For the sake of convenience we recast the above relations in terms of $\hat{R} (= \mathcal{P} R)$

$$\hat{R}_{12}\ \mathcal{P}_{12}\ \eta_{12} L_1^{(\varepsilon_1)}\ \eta_{12}\ L_2^{(\varepsilon_2)} = L_2^{(\varepsilon_2)}\ \eta_{12}\ L_1^{(\varepsilon_1)}\ \eta_{12}\ \hat{R}_{12}\ \mathcal{P}_{12} \tag{10}$$

where $(\eta_{12})^{ab}_{cd} = (-1)^{ab}\ \delta^a_c \delta^b_d$ is a diagonal phase factor. In (10) $\hat{R}_{12}$ is graded and the tensor products L_1 and L_2 remain ungraded (i.e. without any phase factors), contrary to

(9) in which R_{12} matrix is ungraded (i.e. R_{21} is graded) and L_1 and L_2 are graded and are defined in the same fashion as T_1 and T_2.

The pairing $\langle T^i_j, {L^{(+)}}^k_\ell \rangle = R^{ik}_{j\ell}$, $\langle T^i_j, {L^{(-)}}^k_\ell \rangle = {R^{(-1)}}^{ki}_{\ell j}$ puts a restriction on the number of independent generators. We shall list down here the commutation relations (obtained from (10)) involving only the independent generators ${L^{(+)}}^1_2, {L^{(+)}}^2_3, {L^{(-)}}^2_1, {L^{(-)}}^3_2$ and the diagonal elements ${L^{(+)}}^a_a$:

$$
\begin{aligned}
&\left[{L^{(\pm)}}^a_a, {L^{(\pm)}}^b_b\right] = 0 \qquad a, b = 1,2,3,4\\
&{L^{(+)}}^{\binom{a}{b}}_{\binom{a}{b}}\ {L^{(+)}}^1_2 = (p)^{\binom{+1}{-1}}\ {L^{(+)}}^1_2 {L^{(+)}}^{\binom{a}{b}}_{\binom{a}{b}} \qquad a = 1,2 \quad b = 3,4\\
&{L^{(-)}}^{\binom{a}{b}}_{\binom{a}{b}}\ {L^{(+)}}^1_2 = (q)^{\binom{-1}{+1}}\ {L^{(+)}}^1_2\ {L^{(-)}}^{\binom{a}{b}}_{\binom{a}{b}} \qquad {L^{(+)}}^{\binom{a}{b}}_{\binom{a}{b}}\ {L^{(-)}}^2_1 = (p)^{\binom{-1}{+1}}\ {L^{(-1)}}^2_1\ {L^{(+)}}^{\binom{a}{b}}_{\binom{a}{b}}\\
&{L^{(-)}}^{\binom{a}{b}}_{\binom{a}{b}}\ {L^{(-)}}^2_1 = (q)^{\binom{+1}{-1}}\ {L^{(-)}}^2_1\ {L^{(-)}}^{\binom{a}{b}}_{\binom{a}{b}} \qquad {L^{(\pm)}}^{\binom{1}{4}}_{\binom{1}{4}}\ {L^{(+)}}^2_3 = \left(\frac{q}{p}\right)^{\binom{+1}{-1}}\ {L^{(+)}}^2_3\ {L^{(\pm)}}^{\binom{1}{4}}_{\binom{1}{4}}\\
&{L^{(\pm)}}^{\binom{2}{3}}_{\binom{2}{3}}\ {L^{(+)}}^2_3 = (pq)^{\binom{-1}{+1}}\ {L^{(+)}}^2_3\ {L^{(\pm)}}^{\binom{2}{3}}_{\binom{2}{3}} \qquad {L^{(\pm)}}^{\binom{1}{4}}_{\binom{1}{4}}\ {L^{(-)}}^3_2 = \left(\frac{q}{p}\right)^{\binom{-1}{+1}}\ {L^{(-)}}^3_2\ {L^{(\pm)}}^{\binom{1}{4}}_{\binom{1}{4}}\\
&{L^{(\pm)}}^{\binom{2}{3}}_{\binom{2}{3}}\ {L^{(-)}}^3_2 = (pq)^{\binom{+1}{-1}}\ {L^{(-)}}^3_2\ {L^{(\pm)}}^{\binom{2}{3}}_{\binom{2}{3}} \qquad \left({L^{(+)}}^1_2\right)^2 = 0 = \left({L^{(-)}}^2_1\right)^2\\
&\left\{{L^{(+)}}^1_2, {L^{(-)}}^2_1\right\}_{p/q} = \left((pq)^{1/2} - (pq)^{-1/2}\right)\left({L^{(-)}}^2_2\ {L^{(+)}}^1_1 - {L^{(+)}}^2_2\ {L^{(-)}}^1_1\right)\\
&\left[{L^{(+)}}^2_3, {L^{(-)}}^3_2\right] = \left((pq) - (pq)^{-1}\right)\left({L^{(-)}}^3_3\ {L^{(+)}}^2_2 - {L^{(+)}}^3_3\ {L^{(-)}}^2_2\right)
\end{aligned}
\tag{11}
$$

where ${L^{(\pm)}}^{\binom{1}{4}}_{\binom{1}{4}}\ {L^{(+)}}^2_3 = (q/p)^{\binom{+1}{-1}}\ {L^{(+)}}^2_3\ {L^{(\pm)}}^{\binom{1}{4}}_{\binom{1}{4}}$ implies ${L^{(\pm)}}^1_1 {L^{(+)}}^2_3 = (q/p){L^{(+)}}^2_3\ {L^{(\pm)}}^1_1$ and ${L^{(\pm)}}^4_4\ {L^{(+)}}^2_3 = (q/p)^{-1}{L^{(+)}}^2_3 {L^{(\pm)}}^4_4$ and so on. The anticommutator $\{,\}_{p/q}$ is defined as [4]

$$
\{A, B\}_{p/q} = \left(\frac{p}{q}\right)^{1/2} AB + \left(\frac{p}{q}\right)^{-1/2} BA\,. \tag{12}
$$

These generators play the role of quantum analogue of the Cartan–Weyl basis. We notice that ${L^{(+)}}^1_2$ and ${L^{(-)}}^2_1$ are fermionic in nature whereas ${L^{(+)}}^2_3$ and ${L^{(-)}}^3_2$ are bosonic.

The above set of deformed (super-) commutation relations (11) are supplemented by the conditions

$$
L^{(\pm)}\ C^{\mathrm{st}}\ L^{(\pm)\mathrm{st}}(C^{-1})^{\mathrm{st}} = C^{\mathrm{st}}\ L^{(\pm)st}(C^{-1})^{\mathrm{st}}\ L^{(\pm)} = I \tag{13}
$$

which yield

$$
{L^{(+)}}^1_1\ {L^{(+)}}^4_4 = 1 = {L^{(-)}}^1_1\ {L^{(-)}}^4_4 \qquad {L^{(+)}}^2_2\ {L^{(+)}}^3_3 = 1 = {L^{(-)}}^2_2\ {L^{(-)}}^3_3\,. \tag{14}
$$

Equation (13) can be regarded as the dual of (4). It is transparent that the relations (14) belong to the centre of the algebra.

We shall now give the action of the structure maps on the generators. The coproduct $\Delta(L^{(\pm)}) = L^{(\pm)}\dot{\otimes}L^{(\pm)}$ is given by

$$
\begin{aligned}
\Delta\left({L^{(\pm)}}^a_a\right) &= {L^{(\pm)}}^a_a \otimes {L^{(\pm)}}^a_a \qquad a = 1, \ldots, 4\\
\Delta\left({L^{(+)}}^1_2\right) &= {L^{(+)}}^1_1 \otimes {L^{(+)}}^1_2 + {L^{(+)}}^1_2 \otimes {L^{(+)}}^2_2\\
\Delta\left({L^{(+)}}^2_3\right) &= {L^{(+)}}^2_2 \otimes {L^{(+)}}^2_3 + {L^{(+)}}^2_3 \otimes {L^{(+)}}^3_3\\
\Delta\left({L^{(-)}}^2_1\right) &= {L^{(-)}}^2_1 \otimes {L^{(-)}}^1_1 + {L^{(-)}}^2_2 \otimes {L^{(-)}}^2_1\\
\Delta\left({L^{(-)}}^3_2\right) &= {L^{(-)}}^3_2 \otimes {L^{(-)}}^2_2 + {L^{(-)}}^3_3 \otimes {L^{(-)}}^3_2\,.
\end{aligned}
\tag{15}
$$

The co-unit $\varepsilon\left(L^{(\pm)}{}^{i}_{j}\right)$ yields

$$\varepsilon\left(L^{(\pm)}{}^{a}_{a}\right) = 1 \quad \text{and zero for all others}. \tag{16}$$

The antipode S for the orthosymplectic superalgebras can be defined as

$$S(L^{(\pm)}) = C^{\mathrm{st}}\, L^{(\pm)\mathrm{st}}(C^{-1})^{\mathrm{st}} \tag{17}$$

which follows from (13). This gives

$$\begin{array}{lll} S\left(L^{(\pm)}{}^{1}_{1}\right) = L^{(\pm)}{}^{4}_{4} & S\left(L^{(+)}{}^{1}_{2}\right) = \mathrm{i}L^{(+)}{}^{3}_{4} & S\left(L^{(+)}{}^{2}_{3}\right) = -pq\ L^{(+)}{}^{2}_{3} \\ S\left(L^{(\pm)}{}^{2}_{2}\right) = L^{(\pm)}{}^{3}_{3} & S\left(L^{(-)}{}^{2}_{1}\right) = \mathrm{i}L^{(-)}{}^{4}_{3} & S\left(L^{(-)}{}^{3}_{2}\right) = -(pq)^{-1}\ L^{(-)}{}^{3}_{2}. \end{array} \tag{18}$$

The above maps endow the quantum Lie superalgebra with a non-commutative and non-co-commutative Hopf (super-) algebra structure. We have thus obtained a two-parameter quantum universal enveloping superalgebra $U_{p,q}(osp(2/2))$.

The essential task now is to express these results of the R-matrix approach into some standard known form so as to make contact with the classical case. The Lie superalgebra $osp(2/2)$ [10] is of rank 2 and has 6 generators ($H_i, x_i^+, x_i^-, i = 1, 2$) corresponding to the two simple roots. We make the following identification between the two sets of generators:

$$\begin{array}{l} L^{(+)}{}^{1}_{2}\left(L^{(-)}{}^{1}_{1}\ L^{(-)}{}^{2}_{2}\right)^{1/2} = \left((pq)^{1/2} - (pq)^{-1/2}\right)\ x_1^+ \\ L^{(-)}{}^{2}_{1}\left(L^{(-)}{}^{1}_{1}\ L^{(-)}{}^{2}_{2}\right)^{-1/2} = -\left((pq)^{1/2} - (pq)^{-1/2}\right)\ x_1^- \\ L^{(+)}{}^{2}_{3} = \left((pq) - (pq)^{-1}\right)\ x_2^+ \qquad L^{(-)}{}^{3}_{2} = -\left((pq) - (pq)^{-1}\right)\ x_2^-. \end{array} \tag{19}$$

If we define the logarithms H_1, H_2 by

$$\begin{array}{llll} L^{(+)}{}^{1}_{1} = q^{-H_1/2-H_2}\ p^{-H_1/2} & L^{(-)}{}^{1}_{1} = q^{H_1/2}\ p^{H_1/2+H_2} & & \\ L^{(+)}{}^{2}_{2} = p^{-H_2} & L^{(-)}{}^{2}_{2} = q^{H_2} & L^{(+)}{}^{3}_{3} = p^{H_2} & L^{(-)}{}^{3}_{3} = q^{-H_2} \end{array} \tag{20}$$

where

$$q = \mathrm{e}^{h} \qquad p = \mathrm{e}^{h'} \tag{21}$$

then we obtain the following commutation relations between the new set of generators:

$$\begin{array}{l} [H_1, x_1^+] = 2\left(\dfrac{h'-h}{h+h'}\right)\ x_1^+ \qquad [H_1, x_1^-] = 2\left(\dfrac{h-h'}{h+h'}\right)\ x_1^- \\ [H_1, x_2^+] = -4\dfrac{h'}{h+h'}\ x_2^+ = 2a_{12}\left(\dfrac{h'}{h+h'}\right)\ x_2^+ \\ [H_1, x_2^-] = 4\dfrac{h'}{h+h'}\ x_2^- = -2a_{12}\left(\dfrac{h'}{h+h'}\right)\ x_2^- \\ [H_2, x_1^+] = -x_1^+ = a_{21}\ x_1^+ \qquad [H_2, x_1^-] = +x_1^- = -a_{21}\ x_1^- \\ [H_2, x_2^+] = 2x_2^+ = a_{22}\ x_2^+ \qquad [H_2, x_2^-] = -2x_2^- = -a_{22}\ x_2^- \qquad [H_1, H_2] = 0 \\ \{x_1^+, x_1^-\}_{p/q} = \dfrac{(pq)^{H_1/2} - (pq)^{-H_1/2}}{(pq)^{1/2} - (pq)^{-1/2}} \equiv [H_1]_{(pq)^{1/2}} \\ [x_2^+, x_2^-] = \dfrac{(pq)^{H_2} - (pq)^{-H_2}}{(pq) - (pq)^{-1}} \equiv [H_2]_{pq} \end{array} \tag{22}$$

where $\mathcal{A}(a_{ij})$ is the Cartan matrix for $osp(2/2)$ given by

$$\mathcal{A} = \begin{pmatrix} 0 & -2 \\ -1 & 2 \end{pmatrix} \tag{23}$$

which is symmetrized by the matrix $\mathcal{D} = \mathrm{diag}(1,2)$. These generators $(H_i, x_i^{\pm})$ act as q-analogues of Chevalley basis. It is, however, more convenient to introduce the elements

$$k_1 = (pq)^{H_1/2} \qquad k_2 = (pq)^{H_2}\,. \tag{24}$$

Now the super-commutation relations can be written down explicitly as

$$\begin{aligned}
&k_1\, x_1^{\pm} = \left(\frac{p}{q}\right)^{\binom{+1}{-1}} x_1^{\pm}\, k_1 \qquad k_2\, x_1^{\pm} = (pq)^{\binom{-1}{+1}}\, x_1^{\pm}\, k_2 \\
&k_1\, x_2^{\pm} = (p)^{\binom{-2}{+2}}\, x_2^{\pm}\, k_1 \qquad k_2\, x_2^{\pm} = (pq)^{\binom{+2}{-2}}\, x_2^{\pm}\, k_2 \qquad k_1\, k_2 = k_2\, k_1 \\
&\{x_1^+, x_1^-\}_{p/q} = \frac{k_1 - k_1^{-1}}{(pq)^{1/2} - (pq)^{-1/2}} \qquad [x_2^+, x_2^-] = \frac{k_2 - k_2^{-1}}{(pq) - (pq)^{-1}}\,.
\end{aligned} \tag{25}$$

It is also useful to introduce additional generators x_3^+, x_3^-, k_3 (in complete analogy with the Lie superalgebra) using the q-adjoint operation [11]:

$$x_3^+ \equiv [x_2^+, x_1^+] = q\, x_2^+\, x_1^+ - q^{-1}\, x_1^+\, x_2^+ \qquad x_3^- \equiv [x_1^-, x_2^-] = q^{-1}\, x_1^-\, x_2^- - p\, x_2^-\, x_1^-\,. \tag{26}$$

Then one obtains

$$\{x_3^-, x_3^+\}_{p/q} = \frac{(q/p)^{H_2}\,(k_3 - k_3^{-1})}{(pq)^{1/2} - (pq)^{-1/2}} \qquad \text{where} \quad k_3 = k_1 k_2 \tag{27}$$

and the quantum analogue of the Serre relations

$$[x_2^+, x_3^+] = 0 \qquad [x_2^-, x_3^-] = 0 \qquad [x_1^+, [x_1^+, x_3^+]] = 0 \qquad [x_1^-, [x_1^-, x_3^-]] = 0\,. \tag{28}$$

Notice the strangeness in the form of x_3^+ which involves only one deformation parameter and x_3^- which has both p and q.

We have thus arrived at a two-parameter deformation of the universal enveloping algebra $U\,osp(2/2)$. When $p = q \neq 1$, this coincides with the results obtained in [5, 6] for a single parameter, and in the limit $p = q = 1$ we recover the classical Lie superalgebra $osp(2/2)$.

The expressions for the coproduct, co-unit and antipode for the new generators assume the following form.

Co-product

$$\begin{aligned}
&\Delta(k_i) = k_i \otimes k_i \qquad i = 1,2,3 \\
&\Delta(x_1^+) = x_1^+ \otimes k_1^{1/2}\left(\frac{q}{p}\right)^{H_2/2} + k_1^{-1/2}\left(\frac{q}{p}\right)^{-H_2/2} \otimes x_1^+ \\
&\Delta(x_1^-) = x_1^- \otimes k_1^{1/2}\left(\frac{q}{p}\right)^{-H_2/2} + k_1^{-1/2}\left(\frac{q}{p}\right)^{H_2/2} \otimes x_1^- \\
&\Delta(x_2^+) = x_2^+ \otimes k_2\, q^{-H_2} + k_2^{-1}\, q^{H_2} \otimes x_2^+ \qquad \Delta(x_2^-) = x_2^- \otimes k_2\, p^{-H_2} + k_2^{-1}\, p^{H_2} \otimes x_2^- \\
&\Delta(x_3^+) = k_3^{-1/2} \otimes x_3^+ + x_1^+ p^{-H_2} \otimes x_2^+\, k_2^{1/2}\left(\frac{q}{p}\right)^{H_2/2} + x_3^+ \otimes k_3^{1/2} \\
&\Delta(x_3^-) = k_3^{-1/2}\left(\frac{q}{p}\right)^{H_2} \otimes x_3^- + x_2^-\, k_1^{-1/2}\left(\frac{q}{p}\right)^{3H_2/2} \otimes x_1^-\, q^{H_2}\left(\frac{q}{p}\right)^{H_2} + x_3^- \otimes k_3^{1/2}\left(\frac{q}{p}\right)^{H_2}.
\end{aligned} \tag{29}$$

Co-unit

$$\varepsilon(1) = 1 \qquad \varepsilon(k_i) = 1 \qquad \varepsilon(x_i^{\pm}) = 0 \qquad i = 1,2,3\,. \tag{30}$$

Antipode (co-inverse)

$$S(k_i) = k_i^{-1} \qquad i = 1, 2, 3$$
$$S(x_1^+) = -k_1^{1/2} \left(\frac{q}{p}\right)^{H_2/2} x_1^+ \left(\frac{q}{p}\right)^{-H_2/2} k_1^{-1/2}$$
$$S(x_1^-) = -k_1^{1/2} \left(\frac{q}{p}\right)^{-H_2/2} x_1^- \left(\frac{q}{p}\right)^{H_2/2} k_1^{-1/2}$$
$$S(x_2^+) = -k_2\, q^{-H_2}\, x_2^+\, q^{H_2}\, k_2^{-1} \qquad S(x_2^-) = -k_2\, p^{-H_2}\, x_2^-\, p^{H_2}\, k_2^{-1} \tag{31}$$
$$S(x_3^+) = -k_3^{1/2} x_3^+ k_3^{-1/2} + p^{H_2} k_1^{1/2} \left(\frac{q}{p}\right)^{H_2/2} x_1^+ \left(\frac{q}{p}\right)^{-H_2/2} k_1^{-1/2} x_2^+ k_1^{1/2} \left(\frac{q}{p}\right)^{H_2/2} k_3^{-1/2}$$
$$S(x_3^-) = -k_3^{1/2} \left(\frac{q}{p}\right)^{-H_2} x_3^- \left(\frac{q}{p}\right)^{-H_2} k_3^{-1/2} + k_1^{1/2} \left(\frac{q}{p}\right)^{-3H_2/2} q^{H_2} x_2^- q^{-H_2} x_1^- q^{H_2} k_3^{-1/2}.$$

We conclude with the following remarks. The results obtained for quantum supergroup $OSp(2/2)$ and its corresponding universal enveloping algebra $U_{p,q}$ $(osp(2/2))$ can be generalized to higher dimensions i.e. to $U(osp(2m/2n))$ which would involve many more deformation parameters. It is known that in the case of non-super quantum groups, one can obtain multi-parameter quantum groups by deforming the co-multiplication structure of the single-parameter quantum universal enveloping algebra, with the help of a suitable twist operation [12, 13]. It may also be possible to apply the similar prescription to quantum super-algebras to obtain their multi-parametric versions. The supersymmetric formulae we have presented here ((1), (4), (10) and (13)) are very general and hold for all types of orthosymplectic supergroups and their superalgebras belonging to the B, C and D series. In the future, we wish to study the representation theories of these quantum superalgebras.

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